

MATRIX THEORY AND HETEROTIC STRINGS ON TORI

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We consider compactifications of the matrix model of M-theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ for $d > 0$, and interpret them as orbifolds of the supersymmetric $U(N)$ Yang-Mills theory on $\mathbf{R} \times \mathbf{T}^{d+1}$. The orbifold group acts both on the gauge group and on the \mathbf{T}^{d+1} , reduces the gauge group to $O(N)$ over $1 + 1$ dimensional fixed-point submanifolds, and breaks half of the supersymmetry. We clarify some puzzling aspects of the gauge anomaly cancellation in the presence of space-time Wilson lines; in general, the Yang-Mills theory requires certain Chern-Simons couplings to supergravity background fields. We discuss the possibility that D8-branes are present as certain matrix configurations in the Yang-Mills theory, and the fundamental fermions emerge as zero modes. Finally, we point out that the correspondence between matrix theory and string theory suggests the existence of a multitude of non-trivial RG fixed points and dualities in orbifold Yang-Mills theories with eight supercharges in various dimensions.

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1. Introduction

Some of the mystery of M-theory has been recently removed – a very interesting proposal for its non-perturbative description is now available [1], suggesting that M-theory could be described in terms of a matrix model. This “matrix theory” proposal works with microscopic degrees of freedom which exhibit the virtues of D-branes that have been so prominent in our understanding of non-perturbative string dynamics, and elevates them to eleven dimensions.

The proposal has already passed a whole set of tests, in particular in the cases that correspond to compactifications with the maximal amount of supersymmetry. Not only is it apparently possible to reconstruct various expected string dualities, but also string perturbation theory in the light cone gauge seems to be embedded in the non-perturbative framework of matrix theory [2,3]. Since the light-cone-gauge perturbation expansion of string theory is known to be Lorentz invariant, this gives additional credibility to the original matrix theory proposal.

One of the next steps is to consider compactifications of M-theory that respect half of the original supersymmetry, and test whether and how matrix theory reproduces the properties of non-perturbative string theory, M-theory and F-theory, expected on the basis of various conjectured dualities among their vacua. In this paper, we will take certain steps in this direction, and will study compactifications of M-theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ in the matrix theory framework.

These orbifold vacua of M-theory are known to correspond to heterotic string theory, which gives some additional motivation for their study in matrix theory. In particular, here are some of the relevant issues:

- (1) The relation between M-theory on manifolds with boundaries and the $E_8 \times E_8$ heterotic string theory [4,5] predicts the emergence of E_8 boundary degrees of freedom in M-theory. One would want to understand better the microscopic origin of these boundary degrees of freedom. In matrix theory, this question is connected with a more general question about the BFSS proposal, namely, what is the general prescription for compactifications. One option, suggested already in the original BFSS paper [1], is that all of the necessary degrees of freedom are already present in the matrix theory Hamiltonian in uncompactified eleven dimensions. On the other hand, it has been argued in the literature that the proper description of the heterotic compactifications requires extra fermions in the fundamental representation of the gauge group to be added by hand [6,7].

- (2) The BFSS matrix theory has been argued [1] to exhibit the holographic property [8]. A more detailed picture of holography would certainly be useful, as it might elucidate the nature of truly microscopic degrees of freedom in matrix theory. This question also seems to require a better understanding of space-time boundaries in M-theory.
- (3) Toroidal compactifications of heterotic string theory are conjecturally related to various compactifications of string theory, M-theory and F-theory [9,10] on K3 surfaces. Such K3 compactifications seem to be rather difficult to understand in matrix theory directly [11,12], and one might hope that understanding the heterotic side of the conjectured dualities in the matrix theory might illuminate some of the difficulties.
- (4) It was pointed out in [13] that matrix theory, in conjunction with the expected string dualities in various dimensions, leads to predictions about the non-perturbative dynamics of supersymmetric Yang-Mills gauge theories in the corresponding dimensions. In the case of toroidal compactifications analyzed in [13], matrix theory typically leads to postdictions rather than predictions, as the non-trivial fixed points predicted by matrix theory are already known from other considerations. We will see in this paper that when applied to the matrix theory compactifications with half of the maximal supersymmetry, the logic of [13] leads to a multitude of non-trivial predictions about the RG behavior in corresponding Yang-Mills gauge theories.

In section 2 of this paper, we analyze matrix theory compactified on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$, and demonstrate that it is described by Yang-Mills gauge theories on orbifolds, or more precisely, orbifold Yang-Mills gauge theories – the action of the orbifold group on the “space-time” of the Yang-Mills theory is combined with an action on the gauge group itself. A similar class of gauge theories on orbifolds has been studied previously in a remotely related context in [14].

In section 3 we clarify certain aspects of the anomaly cancellation mechanism, relevant to the description of general compactifications with Wilson lines. It is argued that for generic compactifications, the Yang-Mills theory contains certain Chern-Simons terms that couple the Yang-Mills degrees of freedom to a non-trivial supergravity background. The presence of this background is responsible for the local cancellation of gauge anomalies, and is needed for the description of Wilson lines in matrix theory.

In section 4, we further study the physics of D8-branes (or more precisely, longitudinally wrapped 9-branes of M-theory) in matrix theory, and suggest that they appear as certain topologically non-trivial Yang-Mills configurations. The extra fermions corresponding

to 0-8 strings do not have to be added by hand on the basis of anomaly cancellation arguments; rather, they appear as zero modes in the D8-brane background. In this scenario, we expect the supergravity background and the Chern-Simons couplings to be generated when non-zero modes in the configuration describing the D8-branes are integrated out.

In section 5, we briefly compare the Yang-Mills theory description of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ with the expected behavior in string theory, and point out that this comparison suggests the existence of a multitude of non-trivial fixed points in the orbifold supersymmetric Yang-Mills theory on $\mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2$.

While this paper was being written, some closely related results were reported in [15]; in particular, it was independently noted in [15] that matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ is described by super Yang-Mills theory on $\mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2$.

2. Matrix Theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ as an Orbifold Yang-Mills Theory

According to the original BFSS proposal [1], M-theory in the infinite momentum frame is described as a system of $N \rightarrow \infty$ “partons,” represented by D0-branes as the natural carriers of longitudinal momentum. The dynamics is summarized in the quantum mechanical Lagrangian with $U(N)$ gauge symmetry,

$$\mathcal{L} = \frac{1}{2g^2} \int dt \operatorname{tr} \left\{ (D_0 X)^2 + \bar{\theta} D_0 \theta - \frac{1}{2} ([X^i, X^j])^2 - \bar{\theta} \Gamma_i [X^i, \theta] \right\}. \quad (2.1)$$

Here the transverse space-time coordinates X^i , $i = 1, \dots, 9$, as well as their superpartners θ (in the **16** of the transverse Lorentz group $SO(9)$) are in the adjoint representation of the gauge group $U(N)$. Their $U(N)$ matrix elements represent the lowest open-string modes connecting the N D0-branes, whose role has thus been promoted to eleven dimensions. Our normalizations are such that the coupling constant $g = \tilde{R}^{3/2}/(2\pi\ell_{11}^3)$; here \tilde{R} is the radius of the longitudinal dimension, and ℓ_{11} is the eleven-dimensional Planck length.

The kinematical light-cone supersymmetries are:

$$\begin{aligned} \delta A_0 &= \frac{1}{2} \bar{\eta} \theta, \\ \delta_\eta X^i &= \frac{1}{2} \bar{\eta} \Gamma^i \theta, \\ \delta_\eta \theta &= -\frac{1}{2} D_0 X^i \Gamma_i \eta - \frac{1}{4} [X^i, X^j] \Gamma_{ij} \eta, \end{aligned} \quad (2.2)$$

and the supersymmetry algebra is closed up to a gauge transformation.

The \mathbf{Z}_2 orbifold vacua of M-theory that we want to understand can be conveniently studied as \mathbf{Z}_2 orbifolds of matrix theory compactified on a torus. Matrix theory on \mathbf{T}^d is described [1,16,17,18] by the maximally supersymmetric $U(N)$ Yang-Mills gauge theory on the dual torus, $\hat{\mathbf{T}}^d$.¹ This result can be easily derived from the original BFSS model [1,16]. \mathbf{T}^d is a \mathbf{Z}^d orbifold of \mathbf{R}^d , and a configuration of k D0-branes on \mathbf{T}^d can be represented as an infinite array of D0-branes on \mathbf{R}^d that respect the \mathbf{Z}^d symmetry. The orbifold group \mathbf{Z}^d is represented on the original $U(N)$ degrees of freedom of (2.1) in terms of commuting unitary matrices U_m , $m = 1, \dots, d$, and the fields of (2.1) are required to satisfy the corresponding periodicity conditions. The periodicity of the torus along the m -th direction is implemented in the following condition,

$$X^i = U_m X^i U_m^{-1} + e_m^i, \quad (2.3)$$

with e_m^i the components of the corresponding lattice vector. This condition is solved by

$$X_i = D_i \equiv \partial_i + A_i, \quad (2.4)$$

where D_i is the covariant derivative along an extra dimension of the Yang-Mills theory. (2.4) is of course nothing but the traditional formula expressing the action of T-duality on the D-brane world-volume fields. This interpretation invokes the extrapolation of certain aspects of the physics of slowly moving D0-branes in weakly coupled string theory, a useful heuristic analogy that will play its role below.

Now we want to describe \mathbf{Z}_2 orbifolds of matrix theory compactifications on \mathbf{T}^{d+1} , where the orbifold group reflects one of the dimensions in \mathbf{T}^{d+1} .² One could start from the original Lagrangian (2.1), and implement the orbifold procedure which contains both the \mathbf{Z}^d that defines the torus and the \mathbf{Z}_2 which acts on one of the dimensions of the torus by reflection. Due to this non-trivial action of \mathbf{Z}_2 on the torus, the full orbifold group is a product of the non-abelian semi-direct product $\mathbf{Z}_2 \ltimes \mathbf{Z}$ with the abelian group \mathbf{Z}^d representing the “transverse” torus,

$$\mathcal{G} = (\mathbf{Z}_2 \ltimes \mathbf{Z}) \times \mathbf{Z}^d. \quad (2.5)$$

¹ From now on, we will drop the hat denoting the dual torus. In all compactifications considered in this paper, the space-like part of the Yang-Mills parameter space is always related to the compact dimensions in space-time by T-duality.

² In this paper, we always choose the \mathbf{Z}_2 orbifold action such that the longitudinal direction of matrix theory is parallel to the space-time boundary.

One can now follow the steps performed in the case of the toroidal compactification, and factorize the BFSS theory by the non-Abelian discrete group (2.5).

Alternatively, one can perform the orbifold procedure in two steps, and consider \mathbf{Z}_2 orbifolds of the $d + 2$ dimensional $U(N)$ Yang-Mills theory that describes matrix theory compactified on \mathbf{T}^{d+1} . In this paper, we will mostly follow the latter strategy. First, however, let us make the “one-step” approach a little more explicit (for a related discussion, see [19,7,6]).

The translations by elements of \mathbf{Z}^{d+1} have been represented by the commuting unitary matrices U_m . To fully specify the orbifold group (2.5) action in matrix theory, we choose a unitary matrix Ω that represents the generator of \mathbf{Z}_2 . One can use the extrapolation from weakly coupling string theory to argue that Ω will act on the $U(N)$ adjoint fields by a matrix transposition. In Type IIA theory, the \mathbf{Z}_2 of interest is an orientifold symmetry; in particular, it changes the orientation of all open strings. In matrix theory, the $U(N)$ matrix elements correspond to the lowest states of open strings stretching between D0-branes, and the change of world-sheet orientation will transpose the $U(N)$ matrices. Thus, we define

$$\Omega M \Omega = M^T \quad (2.6)$$

for any matrix M in the adjoint of $U(N)$. The full action of Ω on the fields of the Yang-Mills multiplet in addition contains the space-time \mathbf{Z}_2 reflection

$$X^1 \rightarrow -X^1, \quad (2.7)$$

and combines with the representation U_m of the torus translations (2.3) into a representation of the orbifold group (2.5).

Now, consider the spinor fields. On the fermions of eleven-dimensional M-theory, the \mathbf{Z}_2 reflection (2.7) of one of the transverse coordinates acts by [4]

$$\xi \rightarrow \Gamma_1 \xi. \quad (2.8)$$

In the light cone gauge, we impose

$$\Gamma^+ \xi = 0, \quad (2.9)$$

which reduces the 32-component spinor ξ to a 16-component one that we will denote η . We can further choose our gamma matrices such that $\Gamma_1 = \Gamma_2 \dots \Gamma_{10} \Gamma_0 \equiv \frac{1}{2} \Gamma_2 \dots \Gamma_9 [\Gamma_+, \Gamma_-]$, so that the orbifold action (2.8) in the light cone frame becomes

$$\eta \rightarrow \Gamma_2 \dots \Gamma_9 \eta, \quad (2.10)$$

i.e. the fermions are chiral in the transverse eight-dimensional space at the boundary. On the fields of the Yang-Mills theory, which are obtained by dimensionally reducing the ten-dimensional $\mathcal{N} = 1$ theory, this can be rewritten as

$$\eta \rightarrow \Gamma_{01}\eta. \quad (2.11)$$

In terms of the full Yang-Mills field content, we end up with the following orbifold conditions,

$$\begin{aligned} X_1 &= -\Omega X_1 \Omega, & X_i &= \Omega X_i \Omega, \quad i = 2, \dots, 9, \\ A_0 &= -\Omega A_0 \Omega, & \theta &= -\Omega (\Gamma_{01}\theta) \Omega. \end{aligned} \quad (2.12)$$

The action of the orbifold group on the fields induces an action on the symmetries of the original Lagrangian:

$$\begin{aligned} Q &\rightarrow \Gamma_{01}Q, \\ G &\rightarrow -\Omega G \Omega. \end{aligned} \quad (2.13)$$

Only a subgroup of the original symmetry group will survive the projection; in particular, only one half of the original supersymmetry is preserved in the orbifold theory, and the $U(N)$ gauge symmetry is reduced to $O(N)$.

So far we have projected the theory to its \mathbf{Z}_2 -invariant subsector. Since the orbifold group does not act freely, we might expect that self-consistency will require some extra degrees of freedom corresponding to “twisted sectors.” In [6,7] it was indeed argued that sixteen fermions in the fundamental representation of the gauge group have to be added; the precise structure of these twisted modes has been inferred from the extrapolation of D0-brane dynamics in weakly coupled Type IA theory, where they correspond to the massless modes of the 0-8 strings in the Ramond sector.

2.1. Matrix Theory on $\mathbf{S}^1/\mathbf{Z}_2$

Even though the main focus of this paper is on compactifications on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ for $d > 0$, we first review the simple case of $d = 0$ to set the stage for more general compactifications. More details on the $d = 0$ case can be found in [6,7,20].³

This compactification of matrix theory is described by 1 + 1 dimensional Yang-Mills theory whose gauge group has been reduced by the \mathbf{Z}_2 orbifold condition (2.12) to $O(N)$;

³ The case of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2$ has been further analyzed in considerable detail in recent papers, [15,21].

half of the sixteen original supersymmetries are broken, and the theory enjoys $(0, 8)$ supersymmetry in $1 + 1$ dimensions. The field content of the original $U(N)$ theory has been reduced to

$$A_{0,1}^{[IJ]}, \quad X_i^{(IJ)}, \quad x_i, \quad \theta_\alpha^{[IJ]}, \quad \theta_{\dot{\alpha}}^{(IJ)}, \quad \vartheta_{\dot{\alpha}}. \quad (2.14)$$

Here (IJ) indicates the symmetric traceless representation of $O(N)$, and $x_i, \vartheta_{\dot{\alpha}}$ are the singlet parts of the corresponding $U(N)$ fields; of course, the $1 + 1$ chirality of the fermions is correlated with their space-time chirality.

The Lagrangian of this theory is again that of the dimensionally reduced Yang-Mills theory, with the gauge group representations appropriately reduced from $U(N)$ to $O(N)$,

$$\mathcal{L} = \frac{1}{g^2} \int_{\mathbf{S}^1 \times \mathbf{R}} d^2\sigma \operatorname{tr} \left(-\frac{1}{4} F^2 + \frac{1}{2} (D_\mu X^i)^2 - \frac{1}{4} ([X^i, X^j])^2 + \frac{1}{2} (\partial_\mu x^i)^2 + \text{fermions} \right). \quad (2.15)$$

The gauge coupling constant g^2 and the radius ρ_1 of the \mathbf{S}^1 in the Yang-Mills theory are related to the radius R_1 of the space-time orbifold dimension $\mathbf{S}^1/\mathbf{Z}_2$ by

$$g^2 = \frac{\tilde{R}^2}{R_1 \ell_{11}^3}, \quad \rho_1 = (2\pi)^2 \frac{\ell_{11}^3}{\tilde{R} R_1}. \quad (2.16)$$

The surviving field content (2.14) is chiral, and (2.15) represents an anomalous theory. Under an $O(N)$ gauge transformation, the effective action transforms as

$$\delta\mathcal{W} = 32 \cdot \frac{1}{2\pi} \int \operatorname{tr} (\epsilon F). \quad (2.17)$$

It has been argued [20,7] on precisely this basis that a set of 32 chiral fermions χ_r^I in the fundamental representation of the gauge group $O(N)$ should be added to cancel the anomaly. Their contribution to (2.15) is given by

$$\sum_{r=1}^{32} \int d^2\sigma \left(\chi_r^I \partial_+ \chi_r^I + \chi_r^I A_+^{IJ} \chi_r^J \right), \quad (2.18)$$

and their contribution to $\delta\mathcal{W}$ precisely cancels that of (2.17). It is the origin of these extra fields that we would want to understand better from first principles in matrix theory; we will return to this question in section 4.

Thus, compactification of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2$ leads to a relatively standard, Lorentz invariant $(0, 8)$ supersymmetric field theory in $1 + 1$ dimensions. Most of the field content was determined by the \mathbf{Z}_2 orbifold projection, and the rest had to be added in order to cancel the $1 + 1$ dimensional gauge anomaly of those fields that survived the \mathbf{Z}_2 projection.

Now we will see that the fun actually begins in $2 + 1$ dimensions.

2.2. Matrix Theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$

This vacuum is a \mathbf{Z}_2 orbifold of the compactification of matrix theory on \mathbf{T}^2 , and should therefore be describable as an orbifold of the $2+1$ dimensional $U(N)$ gauge theory on the dual \mathbf{T}^2 .

This orbifold theory cannot be the naive, $2+1$ Lorentz-invariant $O(N)$ gauge theory on \mathbf{T}^2 with $\mathcal{N} = 4$ supersymmetry. One can give several arguments that support this statement:

- (1) In the limit where the space-time \mathbf{S}^1 decompactifies, the Yang-Mills theory undergoes dimensional reduction to $1+1$ dimensions, and should reproduce the $1+1$ dimensional theory reviewed in the previous subsection. In particular, one of the scalars of the $1+1$ dimensional Yang-Mills multiplet should come from the third component of the vector potential in $2+1$ dimensions. However, all scalars that survive the \mathbf{Z}_2 projection in $1+1$ dimensions are in the wrong representation (either symmetric traceless or scalar) of $O(N)$ to become the third component of an $O(N)$ gauge field in $2+1$ dimensions.
- (2) In the same limit, the reduced Yang-Mills theory exhibits $(0, 8)$ supersymmetry in $1+1$ dimensions. The usual Poincaré invariant gauge theory in $2+1$ dimensions would give, upon dimensional reduction, a theory with $(4, 4)$ supersymmetry.
- (3) A closely related space-time argument also indicates that the theory cannot exhibit full $2+1$ dimensional Lorentz invariance. In the limit where the space-time $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$ shrinks to zero volume, we expect matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$ to describe the Type I and heterotic $SO(32)$ string theory. These string vacua are super Poincaré invariant in ten dimensions, and the matrix theory description should exhibit at least an $SO(7)$ subgroup of the transverse $SO(8)$ Lorentz symmetry manifest. In the corresponding compactifications on \mathbf{T}^2 , the $SO(9, 1)$ symmetry of the Yang-Mills theory decomposes as $SO(7) \times SO(2, 1)$, where $SO(7)$ is the Lorentz group in the transverse dimensions, and $SO(2, 1)$ is the Yang-Mills Lorentz group. The sixteen supercharges, in the $\mathbf{16}$ of $SO(9, 1)$, decompose as $(\mathbf{8}, \mathbf{2})$. Since we expect the \mathbf{Z}_2 orbifold to preserve the $SO(7)$ symmetry, it must act trivially on the $\mathbf{8}$. However, the orbifold should break half of the supersymmetry, so it has to project out half of the $\mathbf{2}$ of the $2+1$ dimensional Lorentz group. This is of course incompatible with full $2+1$ dimensional Lorentz invariance.

The correct answer, in agreement with all these observations, can be inferred from “first principles” by implementing the orbifold construction directly on the original $0+1$

dimensional theory (2.1). In terms of the $2 + 1$ dimensional Yang-Mills gauge theory, the orbifold group \mathbf{Z}_2 will act not only as a matrix transposition on the $U(N)$ adjoints, and simultaneously as a reflection of the third dimension in the Yang-Mills parameter space,

$$\Omega(\sigma^0, \sigma^1, \sigma^2) = (\sigma^0, \sigma^1, -\sigma^2). \quad (2.19)$$

On the Yang-Mills one-form $A \equiv A_\mu d\sigma^\mu$ and the rest of the multiplet, the \mathbf{Z}_2 will act by

$$\Omega : \quad \begin{cases} A(\sigma^\mu) \rightarrow -\Omega A(\Omega(\sigma^\mu)) \Omega, \\ X^i(\sigma^\mu) \rightarrow \Omega X^i(\Omega(\sigma^\mu)) \Omega, \\ \theta(\sigma^\mu) \rightarrow -\Omega (\Gamma_{01} \theta(\Omega(\sigma^\mu))) \Omega. \end{cases} \quad (2.20)$$

In particular, the components of the gauge field that survive the projection satisfy

$$\begin{aligned} A_{0,1}^{IJ}(\sigma^0, \sigma^1, \sigma^2) &= -A_{0,1}^{JI}(\sigma^0, \sigma^1, -\sigma^2), \\ A_2^{IJ}(\sigma^0, \sigma^1, \sigma^2) &= A_2^{JI}(\sigma^0, \sigma^1, -\sigma^2), \end{aligned} \quad (2.21)$$

in accord with remark (1) above.

These conditions define a certain Yang-Mills theory on $\mathbf{S}^1 \times \mathbf{S}^1 / \mathbf{Z}_2$. More precisely, they define an “orbifold Yang-Mills gauge theory,” since the orbifold group acts simultaneously on the underlying manifold \mathbf{T}^2 as well as the internal $U(N)$ degrees of freedom. Alternatively, this $2 + 1$ dimensional gauge theory can be viewed as a theory on a manifold with two $1 + 1$ dimensional boundaries, and the boundary conditions determined by the orbifold conditions (2.21). It is described by the $9 + 1$ dimensional $\mathcal{N} = 1$ Yang-Mills Lagrangian, dimensionally reduced to $2 + 1$; in particular, notice that the theory still exhibits all sixteen supersymmetries in the bulk, broken to eight supersymmetries only at the boundaries.

Even though our theory is a gauge theory in $2 + 1$ dimensions, its gauge symmetry is potentially anomalous. The mechanism is precisely analogous to the emergence of gravitational anomaly in eleven-dimensional supergravity on a manifold with boundaries [4,5]. Due to the chiral boundary conditions induced by the orbifold at the $1 + 1$ dimensional boundaries, the effective action exhibits an anomaly supported at the boundaries. Under a gauge transformation $\epsilon(\sigma^\mu)$ that respects the \mathbf{Z}_2 symmetry, the effective action transforms as follows,

$$\delta\mathcal{W} = 16 \cdot \frac{1}{2\pi} \int_{V_1} \text{tr} (\epsilon_1 F) + 16 \cdot \frac{1}{2\pi} \int_{V_2} \text{tr} (\epsilon_2 F). \quad (2.22)$$

Here V_1, V_2 are the two $1 + 1$ dimensional components of the boundary, while $\epsilon_1 = \epsilon|_{V_1}$ and $\epsilon_2 = \epsilon|_{V_2}$ have been used to denote the gauge transformation ϵ evaluated at the two boundary components.

The overall coefficient in (2.22) has been determined as follows. Under gauge transformations that project to $1 + 1$ dimensions, i.e. are independent of the coordinate σ^2 normal to the boundaries, the total anomaly should equal the anomaly in the dimensionally reduced $1 + 1$ dimensional theory; in the $2 + 1$ dimensional theory, therefore, each boundary component supports one half of the $1 + 1$ dimensional gauge anomaly (2.17).

This anomaly can be canceled by adding chiral degrees of freedom at each boundary component. A natural choice would be to split the fermions of (2.18) into two groups of sixteen, and place each group at one component of the boundary. We will see in sections 3 and 4 that the actual anomaly-cancellation mechanism is a little more sophisticated.

2.3. Matrix Theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$

Now we can easily generalize our description to $d > 1$. To simplify our discussion, we assume throughout the paper that $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ carries a rectangular metric, and we also set all theta angles corresponding to antisymmetric background fields to zero; generalizations to generic tori are straightforward.

The arguments of the previous subsection can be repeated to show that each additional dimension compactified on a space-time circle gives rise to a dimension in the Yang-Mills parameter space which is odd under the \mathbf{Z}_2 action. We will denote the corresponding coordinates by $\sigma^\mu = (\sigma^0, \sigma^1, \dots, \sigma^{d+1})$. The orbifold group acts on the Yang-Mills parameter space by Ω ,

$$\Omega : \begin{cases} \sigma^\mu \rightarrow \sigma^\mu, & \mu = 0, 1, \\ \sigma^\mu \rightarrow -\sigma^\mu, & \mu = 2, \dots, d+1. \end{cases} \quad (2.23)$$

The singular locus of this \mathbf{Z}_2 action has 2^d components, each of them being a $1 + 1$ dimensional cylinder $\mathbf{S}^1 \times \mathbf{R}$. The full action of the orbifold group on the Yang-Mills multiplets is again given by (2.20).

Thus, we obtain the following correspondence,

$$(\text{matrix theory on } \mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d) \longleftrightarrow (\text{Yang Mills theory on } \mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2). \quad (2.24)$$

In this correspondence, the supersymmetric Yang-Mills theory on $\mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2$ is such that the orbifold group acts simultaneously on the Yang-Mills parameter space and on the

gauge group representations. The theory is the maximally supersymmetric, $U(N)$ Yang-Mills gauge theory in the bulk, and is described by the dimensionally reduced Yang-Mills Lagrangian,

$$\mathcal{L} = \frac{1}{g^2} \int d^{d+2} \sigma \operatorname{tr} \left(-\frac{1}{4} F^2 + \frac{1}{2} (D_\mu X^i)^2 - \frac{1}{4} ([X^i, X^j])^2 + \text{fermions} \right). \quad (2.25)$$

Over the $1+1$ dimensional manifolds of fixed points, the gauge symmetry is reduced to $O(N)$, and one half of the sixteen supersymmetries are broken. From the point of view of the $1+1$ dimensional singular locus, the boundary conditions are chiral; fields that are even under the \mathbf{Z}_2 symmetry are precisely the multiplets (2.14) of the $1+1$ dimensional theory, and the eight unbroken supersymmetries all carry the same $1+1$ dimensional chirality.

Several comments are in order:

- (1) It is of course not surprising to see the role of \mathbf{S}^1 and $\mathbf{S}^1/\mathbf{Z}_2$ interchanged as we switch from the space-time to the Yang-Mills description of the theory according to (2.24). These two descriptions are related by T-duality, and it is a well-known property of T-duality in open string models that it exchanges \mathbf{S}^1 with $\mathbf{S}^1/\mathbf{Z}_2$ [22].⁴
- (2) Gauge theories of this orbifold type are not really new; their close relatives were studied in a context related to D-branes and to open string vacua in [14]. It was shown in [14] that Chern-Simons gauge theories on $2+1$ dimensional orbifolds are related to conformal field theories on surfaces with boundaries and/or crosscaps, and provide a natural framework for the study of orientifold and D-brane conformal field theories. The orbifold gauge theories of [14] exhibit some striking similarities with the theories we encounter in the matrix model correspondence (2.24).
- (3) The Yang-Mills coupling constant g^2 and the radii ρ_m of the Yang-Mills parameter space $\mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2$ are related to the radii R_m of the space-time $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ by

$$g^2 = (2\pi)^{2d} \ell_{11}^{3d-3} \frac{\tilde{R}^{2-d}}{R_1 \dots R_{d+1}}, \quad \rho_m = (2\pi)^2 \frac{\ell_{11}^3}{\tilde{R} R_m}. \quad (2.26)$$

In cases other than $d=2$, the coupling constant g^2 is dimensionful, and can be rescaled to give

$$\tilde{g}^2 = \frac{g^2}{(\rho_1 \dots \rho_{d+1})^{(d-2)/(d+1)}} = (2\pi)^4 \frac{\ell_{11}^3}{(R_1 \dots R_{d+1})^{3/(d+1)}}. \quad (2.27)$$

⁴ The orbifold geometry $\mathbf{S}^1 \times \mathbf{T}^d/\mathbf{Z}_2$ has also been encountered in a similar context by Kutasov, Martinec and O'Loughlin in their approach to non-perturbative M-theory [23]. I am grateful to Martin O'Loughlin for discussions on the results of [23].

Let us return to the orbifold Yang-Mills theory of (2.24). This theory is potentially anomalous, with the anomaly supported at the planes of fixed points. Just as in the case of $d = 1$, we find that under a gauge transformation, the effective action transforms as follows,

$$\delta\mathcal{W} = \frac{1}{2^d} \cdot \frac{16}{\pi} \sum_{\alpha} \int_{V_{\alpha}} \text{tr} (\epsilon F). \quad (2.28)$$

(Here $\alpha = 1, \dots, 2^d$ parametrizes the components of the singular locus.) The overall coefficient has again been determined by symmetry arguments similar to those of [4,5]; each plane of fixed points supports an anomaly equal to 2^{-d} times the anomaly of the $1 + 1$ dimensional theory. We can try to cancel this anomaly by adding chiral degrees of freedom to the theory, the obvious choice being a set of fermions described by

$$\sum_{\alpha} \int_{V_{\alpha}} d^2\sigma \text{tr} \chi(\partial_+ + A_+)\chi, \quad (2.29)$$

a prescription which can only work if the number of space-time dimensions compactified on \mathbf{T}^d is lower than six. Another troubling situation would occur in the presence of space-time Wilson lines on \mathbf{T}^d for any d ; the natural Yang-Mills description of such vacua would require the fermions to be located away from V_{α} , and anomalies could not be cancelled locally by the suggested mechanism. These arguments indicate that our picture of anomaly cancellation still misses some crucial ingredients. We will present a solution to this puzzle in section 3.

2.4. Comparison to Weak String Coupling

Resorting once again to the extrapolation into the regime of slowly moving D-branes in weakly coupled string theory, we can argue that precisely this orbifold Yang-Mills theory could have been expected to appear in the description of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$.

Consider a system of N D0-branes in Type IA string theory on a torus, close to one of the orientifold planes. In order to keep the string coupling small everywhere, we put sixteen D8-branes on top of each orientifold plane. By T-duality in the orbifold dimension, this system is mapped to a system of N D1-branes in Type I theory; the orientifold plane becomes space-filling, and intersects each D1-string along the $1 + 1$ dimensional manifold which is the string itself. By another T-duality, now in one of the dimensions parallel to the original D8-branes, this configuration is again mapped to Type IA theory, where it represents N D2-branes ending on two $8 + 1$ dimensional orientifold planes. This

can be further dualized along any of the extra dimensions parallel to the original D8-branes, leading to a hierarchy of systems in successive dimensions representing N D p -branes intersecting 2^{p-1} orientifold planes of dimension $11 - p$ along 2^{p-1} strings. The Yang-Mills description of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ in terms of the orbifold gauge theory corresponds to the world-volume gauge theory of precisely this configuration of branes intersecting orientifold planes (and the D(10 - p)-branes T-dual to the original D8-branes) for $d = p - 1$, and extrapolated into the regime of strong string coupling.

3. Anomaly Cancellation and Chern-Simons Terms in Matrix Theory

The simple matrix theory description of compactifications on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ that we obtained in the previous section seems to run into problems in the cases that do not permit local cancellation of anomalies by distributing the fermions among the components of the singular locus. Such configurations would appear for d higher than five, but also in any dimension in the presence of generic Wilson lines around the space-time torus \mathbf{T}^d .

In this section, we present a resolution which is very close in spirit to the resolution of similar puzzles in orientifold compactifications of string theory and M-theory, cf. e.g. [24]. While this argument will indeed show that the theory is non-anomalous, we will see indications that the suggested resolution should perhaps be interpreted as an effective description of a more microscopic mechanism; we will indeed suggest a more microscopic picture in section 4.

In the previous section, we have seen that the orbifold Yang-Mills theory, when extrapolated to the regime of weak string coupling, corresponds to the world-volume gauge theory of branes intersecting other branes and orientifold planes along strings. In that extrapolated context, it is known that no anomalies appear [25,24,26]; the gauge theory on the brane world-volume contains a topological coupling to the RR background fields [26],

$$\int C \wedge \text{tr } e^F, \quad (3.1)$$

and the orientifold planes (as well as the branes that intersect the world-volume) carry a non-zero RR charge. Both the orientifold planes and the intersecting branes are separately non-anomalous, because the would-be anomalies are cancelled locally due to the anomaly inflow from the bulk mediated by (3.1).

Assuming no surprises happen as we extrapolate to the regime of strong string coupling, this mechanism resolves our puzzle, at the cost of introducing the explicit Chern-Simons couplings to σ -dependent RR backgrounds.⁵ We postulate that the fixed-point planes V_α carry a RR charge, and add the explicit Chern-Simons terms (3.1) to the matrix theory Lagrangian; the local anomaly cancellation is then ensured by the anomaly inflow from the bulk of the Yang-Mills parameter space.

Let us study the suggested mechanism in more detail. Since our focus is on infrared effects, it is natural to consider the 9+1 dimensional Yang-Mills theory; the cases describing uncompactified space-time dimensions can be recovered from this theory by dimensional reduction. In this context, each component V_α of the fixed-point locus is an electric source for a RR 2-form C' , and a magnetic source for the RR 6-form C . In particular, C will satisfy the modified Bianchi identity,

$$dG = e_\alpha \delta_{V_\alpha}, \quad (3.2)$$

where G is the field strength of C and δ_{V_α} is the delta function localized at the $1+1$ dimensional submanifold V_α (and interpreted as a d -form), and e_α is the charge of V_α . This charge can be determined by extrapolating from the weakly-coupled regime [26,24]; in the case that corresponds to matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$, each individual component V_α of the singular locus carries -2^{5-d} units of the corresponding D-brane charge.

The relevant part of (3.1) is then

$$- \int G \wedge \text{tr } \omega_3(A), \quad (3.3)$$

with the Chern-Simons three-form defined by $d\omega_3(A) = F \wedge F$, and

$$- \int \star G \wedge \text{tr } \omega_7(A), \quad (3.4)$$

where the Chern-Simons seven-form satisfies $d\omega_7(A) = F \wedge F \wedge F \wedge F$. The first term modifies the anomaly structure of the Yang-Mills gauge symmetry, and is responsible for the local anomaly cancellation near the fixed-point submanifolds V_α . Under a gauge transformation, we find

$$\begin{aligned} \delta \left(- \int G \wedge \text{tr } \omega_3(A) \right) &= - \int G \wedge \text{tr } (D\epsilon \wedge F) \\ &= \int dG \wedge \text{tr } (\epsilon F) = \sum_\alpha e_\alpha \int_{V_\alpha} \text{tr } (\epsilon F), \end{aligned} \quad (3.5)$$

⁵ Even though these background fields now belong to the multiplet of eleven-dimensional supergravity, we will frequently refer to them using the ten-dimensional string language.

which is precisely the result we need to cancel the anomalies locally at V_α .

The second Chern-Simons term, (3.4), measures the D8-brane charge carried by the corresponding configuration. There are two sources of the D8-brane charge in the theory: the fixed point submanifolds V_α that correspond to the orientifold planes, and the locations of the fermions χ corresponding to the D8-branes themselves. To ensure the global existence of G , the total background charge must add up to zero. Even though the fermions are no longer needed for anomaly cancellation, their presence is still required by the background charge conservation.

The configurations with a non-democratic distribution of fermions among the fixed-point planes – corresponding to vacua with generic space-time Wilson lines – are not anomalous; they do require, however, a non-zero RR background.⁶ Once there is a non-trivial RR background in the theory, we should expect a non-trivial NS background to accompany it, e.g. on the basis of the space-time equations of motion. It would be very important to understand directly in the Yang-Mills theory how these background fields self-adjust in order to represent consistent backgrounds of matrix theory with the right amount of supersymmetry. Given a consistent background, the Chern-Simons terms (3.1) will receive modifications due to the non-zero curvature and the NS two-form. The complete Chern-Simons couplings in the extrapolated regime of D-brane world-volume field theories at weak string coupling can be found in [26]; in particular, the presence of non-zero curvature will modify (3.1) to [26]

$$\int C \wedge \text{tr } e^F \wedge \sqrt{\hat{A}(R)}. \quad (3.6)$$

Thus, the requirement of local anomaly cancellation leads us to an intricate picture, whereby generic vacua of matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ are described by orbifold Yang-Mills gauge theories with couplings to a non-trivial supergravity background.

3.1. Wilson Lines

We have argued that the Yang-Mills theory is locally non-anomalous in the vicinity of each plane of fixed points, no matter whether this plane supports any of the fermions χ . Turning this argument around, the fermions do not have to be located at the fixed-point planes to cancel anomalies – we can group them in sixteen pairs, and each pair can

⁶ Notice that in general, the background breaks the original sixteen supercharges to eight supersymmetries even in the bulk.

be supported by a separate $1 + 1$ dimensional surface $S_{\tilde{\alpha}}$ ($\tilde{\alpha} = 1, \dots, 16$) parallel to the fixed-point planes V_{α} . Now we will argue that this configuration describes matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ with generic Wilson lines around the \mathbf{T}^d ; the locations of $S_{\tilde{\alpha}}$ are given by the eigenvalues of the space-time Wilson lines.

Consider M-theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ in the presence of space-time Wilson lines. Introducing the following notation,

$$D(\theta) \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (3.7)$$

the Wilson lines W_m ($m = 1, \dots, d$) can always be brought into the following diagonal form,

$$W_m = \text{diag}(D(\theta_{i,1}), \dots, D(\theta_{i,16})). \quad (3.8)$$

In the Yang-Mills theory, this configuration is described as follows. First we group the 32 fermions into sixteen pairs indexed by $\tilde{\alpha} = 1, \dots, 16$. Each pair will be propagating on a $1 + 1$ dimensional surface $S_{\tilde{\alpha}}$ which is parallel to the fixed-point surfaces V_{α} , and located in the space transverse to V_{α} at coordinates

$$(\sigma^2, \dots, \sigma^{d+1}) = (\pi\theta_{1,\tilde{\alpha}}\rho_1, \dots, \pi\theta_{d,\tilde{\alpha}}\rho_d). \quad (3.9)$$

As long as the surfaces $S_{\tilde{\alpha}}$ that support the fermions are located away from the fixed-point surfaces V_{α} , the fermions carry the fundamental representation of $U(N)$. Since the fermions are chiral, the path integral develops a gauge anomaly, now localized at the $1 + 1$ dimensional surfaces $S_{\tilde{\alpha}}$. This anomaly is cancelled by the mechanism of section 3. Recall that each $S_{\tilde{\alpha}}$ carries one unit of the D8-brane charge; therefore, $S_{\tilde{\alpha}}$ contributes to the right hand side of the Bianchi identity (3.2), and the anomaly inflow due to the Chern-Simons coupling ensures that the anomaly cancels locally. Notice that at $S_{\tilde{\alpha}}$, half of the original supersymmetry is broken.

The cases with the democratic distribution of fermions among V_{α} are only possible for $d \leq 5$, and correspond to the Wilson lines that break the gauge group to 2^d copies of $SO(2^{5-d})$. The generic cases are also non-anomalous, but they require a non-zero RR background; this in turn generates a non-trivial gravitational background, and changes the simple picture of parallel $1 + 1$ dimensional surfaces on a flat orbifold.

Similarly, the vacua with generic locations of D8-branes along the space-time orbifold dimension $\mathbf{S}^1/\mathbf{Z}_2$ can be understood in matrix theory as follows. The locations of D8-branes in $\mathbf{S}^1/\mathbf{Z}_2$ translate into the Wilson lines in the D8-brane gauge group around the

\mathbf{S}^1 dimension of the Yang-Mills parameter space. This construction requires an explicit coupling to the D8-brane gauge fields, and generates non-trivial holonomies for the corresponding fermions around the compact dimension of $S_{\tilde{\alpha}}$. The phases of these holonomies are equal to the locations of the corresponding D8-branes along the space-time orbifold dimension in the units of its radius.

3.2. The Chern-Simons Term in $2 + 1$ Dimensions

Before moving on, let us take a closer look at the proposed $2 + 1$ dimensional Chern-Simons term in the case corresponding to matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$. Chern-Simons terms in $2 + 1$ dimensions change their sign under parity, and it is somewhat unusual to suggest their presence in a theory which is projected to the parity-invariant sector. In the case at hand, this question is resolved as follows: the Chern-Simons term manages to be parity-even, because its “coupling constant” is itself odd under parity! Indeed, the Chern-Simons coupling is given by the RR field strength, which in $2 + 1$ dimensions is a zero-form. It is constant everywhere outside the singular locus V_α (and away from the D8-branes at $S_{\tilde{\alpha}}$). However, at V_α (as well as $S_{\tilde{\alpha}}$) it develops a step-function singularity, because V_α (and $S_{\tilde{\alpha}}$) serve as sources in its Bianchi identity. Ignoring the D8-branes for the moment, (3.3) in $2 + 1$ dimensions close to the boundary can be written as

$$\int G \operatorname{tr} \omega_3 = k \int \varepsilon(\sigma_2) \operatorname{tr} \omega_3, \quad (3.10)$$

where $G \equiv k\varepsilon(\sigma_2)$; this indeed makes the Chern-Simons term even under parity.

Notice that this is somewhat reminiscent of certain Chern-Simons terms in $4 + 1$ dimensions, encountered in a closely related context on the world-volume of five-brane probes in Type IA vacua [27]; it also exhibits some similarities with the mechanism that ensures parity invariance in effective M-theory in eleven dimensions despite the presence of its Chern-Simons self-interaction term [28].

4. D8-Branes from Matrices

Although the Chern-Simons couplings to a non-trivial RR background solve the anomaly cancellation problem in the matrix theory description of the potentially anomalous configurations, the explicit presence of a closed-string background is not entirely satisfactory. One of the lessons we have learned in D-brane physics [29,30] is that the long-distance effects of gravity – and more generally, effects connected with a closed-string exchange – are

reinterpreted at substringy scales as open-string effects. While it is conceivable that the matrix theory description of certain vacua may require explicit couplings to closed string backgrounds of the type we encountered in section 3, one might be tempted to interpret such couplings as part of an effective description of a more microscopic mechanism, with the supergravity background being generated when certain heavy matrix modes are integrated out. In this section we will suggest that these extra degrees of freedom are related to the microscopic description of D8-branes in matrix theory.

So far in this paper, the extra fermions χ have been the only indication that the $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$ vacuum of matrix theory contains D8-branes.⁷ We have seen that the fermions χ are not needed to cancel local gauge anomalies at the fixed-point planes V_α ; generically, they are not even located at V_α , and should presumably be thought of as localized bulk degrees of freedom. These observations suggest that the corresponding D8-branes could have an intrinsic description in matrix theory, as localized bulk configurations in the orbifold Yang-Mills theory. In this section we present evidence that this is indeed the case, and demonstrate that D8-branes appear as certain topologically non-trivial configurations in the Yang-Mills theory.

4.1. D8-Branes in Matrix Theory

In the previous section, we have seen that in order to ensure local cancellation of anomalies, the surfaces $S_{\tilde{\alpha}}$ that support the chiral fermions χ had to be postulated to carry one unit of the D8-brane charge. We want to replace this argument by a more microscopic picture, and interpret the fermion as a zero mode in a Yang-Mills background which itself carries the corresponding charge. Consider Yang-Mills configurations with non-zero fourth Chern character,

$$ch_4(F) = \frac{1}{4!(2\pi)^4} \int \text{tr} (F \wedge F \wedge F \wedge F). \quad (4.1)$$

Such configurations couple naturally to the corresponding RR field C' , and carry a non-zero D8-brane charge given by the value of $ch_4(F)$. The configuration with the minimum unit of this charge

$$\frac{1}{4!(2\pi)^4} \int \text{tr} (F \wedge F \wedge F \wedge F) = 1, \quad (4.2)$$

⁷ As in the rest of the paper, we will refer to the longitudinally wrapped 9-branes of M-theory as “D8-branes” for short.

is a natural candidate for the matrix theory description of one D8-brane.⁸ A virtually identical conjecture has appeared in a broader context in [31]; our analysis, based on arguments motivated by the anomaly cancellation mechanism of section 3, provides further support for this conjecture.

In the decompactified case, i.e. in matrix theory on $\mathbf{S}^1/\mathbf{Z}_2$ described by 1 + 1 dimensional Yang-Mills theory, the configurations of matrices that carry non-zero $ch_4(F)$ are translated with the use of (2.4) to configurations of matrices with the corresponding non-zero value of

$$\text{tr } ([X, X] \wedge [X, X] \wedge [X, X] \wedge [X, X]). \quad (4.3)$$

(Here the wedge products indicate that the space-time indices are contracted with the completely antisymmetric tensor in the eight transverse dimensions.) This configuration carries infinite energy, as expected, since it describes a configuration of branes with infinite volume.

How do we construct explicit solutions of the Yang-Mills equations of motion that carry non-zero D8-brane charge (4.1)? One might be tempted to consider some kind of self-duality conditions in eight dimensions, of the type

$$F_{\mu\nu} = T_{\mu\nu\sigma\rho} F^{\sigma\rho}, \quad (4.4)$$

with $T^{\mu\nu\sigma\rho}$ a certain fixed four-form in eight dimensions. A large literature is devoted to this subject (see e.g. [32]). Self-duality in eight dimensions indeed exhibits some remarkable properties: group-theoretical classifications of possible four-forms $T^{\mu\nu\sigma\rho}$ have been presented, and an analogy of the ADHM construction has been shown to exist in some cases; the algebra of octonions plays a prominent role in some of these constructions. In general, however, the corresponding self-dual Yang-Mills configurations break more supersymmetry than we want for the present purpose. While self-duality in eight dimensions as discussed in [32] might still correspond to interesting brane configurations in matrix theory, we will resort to other means that will allow us to construct a D8-brane configuration in matrix theory.

To construct a Yang-Mills configuration with non-zero D8-brane charge (4.1), we will use lower-dimensional brane configurations. Decompose the eight-torus as the product of

⁸ This D8-brane will in general carry stacks of lower-dimensional branes in its world-volume. In order to have a configuration with only the D8-brane charge, we would have to make sure that the corresponding lower-dimensional cohomology classes vanish.

two four-tori, $\mathbf{T}_1 \times \mathbf{T}_2$, and consider two four-dimensional $SU(2)$ Yang-Mills instantons $\mathcal{A}^{(1)}$ and $\mathcal{A}^{(2)}$ with instanton number one on \mathbf{T}_1 and \mathbf{T}_2 respectively. Set

$$\tilde{A} = \mathcal{A}^{(1)} + \mathcal{A}^{(2)}; \quad (4.5)$$

this defines an $SU(2)$ gauge potential on $\mathbf{T}_1 \times \mathbf{T}_2$ that carries one unit of the D8-brane charge (4.1). As in [31], this configuration on $\mathbf{T}_1 \times \mathbf{T}_2$ has no gauge invariant moduli; when translational invariance is broken, for example by the orbifold \mathbf{Z}_2 action on the torus, the configuration will have a gauge invariant modulus, corresponding to the relative location of the instanton with respect to the orbifold singularities.

In addition to the D8-brane charge, the configuration (4.5) will carry D4-brane charges, due to the presence of D4-branes represented by the individual instantons $\mathcal{A}^{(1)}$ and $\mathcal{A}^{(2)}$ in the world-volume of the D8-brane. We have been unable to find a configuration which would only carry the D8-brane charge while preserving the amount of supersymmetry required by the longitudinally wrapped 9-brane of M-theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$.

Modulo this fact, we are now ready to describe a system of D0-branes in the presence of the D8-brane. Assume that the Yang-Mills configuration carrying one unit of the D8-brane charge (4.1) is given by a gauge potential \tilde{A} in an $SU(N_0)$ subgroup of the full gauge group; for simplicity, we will refer to \tilde{A} as the “instanton.” (As an example, one can consider (4.5), in which case $N_0 = 2$ and the “instanton” is constructed from the two four-dimensional $SU(2)$ instantons localized in dimensions $2 \dots 5$ and $6 \dots 9$ respectively.) To describe the background with k D8-branes, we select k mutually commuting $U(N_0)$ subgroups in the Yang-Mills gauge group, and consider the configurations with one unit of D8-brane charge in each of the $U(N_0)$ sectors. The choice of the k subgroups breaks the gauge symmetry of matrix theory to

$$U(N) \times \underbrace{U(N_0) \times \dots \times U(N_0)}_k, \quad (4.6)$$

which is further broken in the $U(N_0)$ sectors by the $SU(N_0)$ gauge backgrounds \tilde{A} . In the background of k instantons, we will decompose the Yang-Mills gauge field as

$$\begin{pmatrix} A & B^1 & \dots & B^k \\ B^{\dagger 1} & \tilde{A}^{(1)} + C^{11} & \dots & C^{1k} \\ \vdots & \vdots & \ddots & \vdots \\ B^{\dagger k} & C^{k1} & \dots & A^{(k)} + C^{kk} \end{pmatrix} \quad (4.7)$$

(and similarly for the fermions). In (4.7), $\tilde{A}^{(p)}$ is the instanton gauge field background in the p -th copy of $U(N_0)$.

According to our conjecture, this configuration describes N D0-branes in the presence of k D8-branes (possibly with lower-dimensional branes in their world-volumes if its lower-dimensional Chern numbers are non-zero [31]). In this scenario, D8-branes appear as composites of “partonic D0-branes,” in accord with the original philosophy of the BFSS proposal [1].

4.2. 0-8 Strings and 8-8 Strings from Collective Matrix Coordinates

In the instanton background (4.7), matrix theory contains the adjoint $U(N)$ degrees of freedom (in the block denoted by A in (4.7)), plus other degrees of freedom due to the presence of the instanton background. We stress that all these degrees of freedom are a part of the original adjoint large- N matrices; the choice of the subgroup that contains the instanton background breaks the original large- N gauge group to (4.6). It is natural to interpret the $U(N)$ group of (4.6) in the large- N limit as the matrix theory gauge group, and derive an effective matrix theory in the presence of the D8-brane degrees of freedom. In this effective theory, in addition to the adjoint $U(N)$ fields, one would keep only the zero modes and integrate out the non-zero modes of the matrix elements that are not in the adjoint of $U(N)$, thus generating an effective description of the theory in terms of $N \rightarrow \infty$ D0-branes that are not “partonic components” of the D8-branes, plus extra degrees of freedom from the zero modes.

With this picture in mind, consider the off-diagonal blocks in (4.7) (and their superpartners).

- (1) The off-diagonal matrices B carry the fundamental representation of the $U(N)$ gauge group of the N D0-branes that do not appear as partons in the D8-branes. Microscopically, the matrix elements in B (and their superpartners) represent 0-0 strings stretching between a given D0-brane and a “parton” within the D8-brane; effectively, they should give rise to the 0-8 strings.
- (2) The $C^{\ell m}$ matrices in (4.7) are $U(N)$ singlets, and for $\ell \neq m$ carry the fundamental of the ℓ -th and m -th copy of $U(N_0)$. Microscopically, they represent open strings stretching between two “partons” in the corresponding two D8-branes, and should therefore be interpreted as the matrix-theory description of the 8-8 strings.

We expect the fermions χ to appear as the low-energy collective excitations in the B sector of (4.7). To check this, we will study the effective $U(N)$ dynamics in the presence

of a non-trivial instanton gauge field \tilde{A} . To be specific, we will again take \tilde{A} to be the instanton of (4.5). Each four-dimensional instanton \mathcal{A} in (4.5) has a chiral fermion zero-mode in the fundamental representation of $U(N)$; hence, the eight-dimensional instanton (4.5) will have a chiral fermion zero mode χ_0 , satisfying

$$\Gamma^\mu D_\mu(\tilde{A})\chi_0 \equiv D(\mathcal{A}^{(1)})\chi_0 + D(\mathcal{A}^{(2)})\chi_0 = 0. \quad (4.8)$$

(Here $D(\mathcal{A})$ is the four-dimensional Dirac operator in the instanton background \mathcal{A} .) This zero mode is chiral in eight dimensions; in the two dimensions (σ^0, σ^1) transverse to the instanton, it gives rise to a chiral $1+1$ dimensional field χ_0 in the fundamental representation of $U(N)$. In the presence of k D8-branes, one gets k chiral $1+1$ dimensional fermions in the fundamental of $U(N)$, and described by the effective action

$$\int_{S_{\tilde{\alpha}}} d^2\sigma \operatorname{tr} \bar{\chi}_0(\partial_+ + A_+)\chi_0, \quad (4.9)$$

precisely as expected on the basis of the suggested relation to the 0-8 strings. The $1+1$ dimensional surfaces $S_{\tilde{\alpha}}$ in (4.9) are at the core of the corresponding instantons.

Similarly, consider the C sector. According to our conjecture, the zero modes should correspond to the massless modes of the 8-8 strings. Masses of the lowest 8-8 string modes are proportional to the distance between the corresponding D8-branes (and depend appropriately on the Wilson lines, should those be present in the vacuum). In particular, when two or more branes coincide, the world-volume gauge symmetry is enhanced. Now we will show that the instanton background (4.7) contains zero modes that exhibit this enhanced symmetry behavior. In the general instanton background, the off-diagonal Yang-Mills gauge fields gain masses, and the gauge symmetry (4.6) is broken to

$$U(N) \times \underbrace{U(1) \times \dots \times U(1)}_k. \quad (4.10)$$

The k copies of $U(1)$ are unbroken as the instantons take values in $SU(N_0)$. However, if the locations of two instantons coincide in the σ space, a new massless mode appears, as we can now rotate the two instantons as rigid objects in $U(2N_0)$. Together with the unbroken $U(1)$'s, this generates the expected $U(2)$. If k instantons coincide, this gives rise to bosonic zero modes

$$\begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & a_{11}I_{N_0} & \dots & a_{1k}I_{N_0} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{k1}I_{N_0} & \dots & a_{kk}I_{N_0} \end{pmatrix}, \quad (4.11)$$

where I_{N_0} is the $N_0 \times N_0$ unit matrix, and

$$\begin{pmatrix} a_{11} & \dots & a_{1k} \\ \vdots & \ddots & \vdots \\ a_{k1} & \dots & a_{kk} \end{pmatrix} \quad (4.12)$$

is in the adjoint representation of $U(k)$. The $U(k)$ transformations can depend on the dimensions σ^μ ($\mu = 0, 1$) transverse to the instanton, and we recover the bosonic sector of the gauge theory on the world-volume of the D8-brane.⁹ This is in accord with the suggested interpretation of these zero modes as the massless 8-8 string modes.

In this section, we have suggested that D8-branes in matrix theory are described by non-trivial matrix configurations. In this scenario, the massless modes of 0-8 strings and 8-8 strings emerge as the zero modes in the corresponding background, and do not have to be artificially added into the theory. We can integrate out the off-diagonal non-zero modes, thus obtaining an effective description in terms of the D0-branes and the extra degrees of freedom from the zero modes. We expect that this should reproduce the Chern-Simons couplings with effective supergravity backgrounds, in the spirit of [29]. This is indeed plausible, since the non-zero modes include degrees of freedom that can be interpreted as massive 0-8 (and 8-8 strings), which are known to be responsible for the effective supergravity background at large distances.

Given that the D8-branes may have an intrinsic description in matrix theory, one wonders whether the orientifold planes can themselves be described in an intrinsic way. They serve as sources of the RR background, which is again indicative of an effective theory. We have nothing to say about this in this paper, except for noting that such a description may not be completely unexpected. Indeed, Sen has shown [33] that in certain orientifold compactifications of Type IIB string theory, the orientifold planes split non-perturbatively in the string coupling constant into a pair of D-branes. In the matrix theory context, such phenomena would indicate that the theory may not respect the orbifold procedure assumed as a part of the kinematics in our construction; the full Yang-Mills theory may no longer be an orbifold theory non-perturbatively.

⁹ More precisely, on the two-dimensional locus $S_{\tilde{\alpha}}$ which corresponds to the D8-brane location in the Yang-Mills parameter space.

5. Orbifold Yang-Mills Dynamics from Matrix Theory

Having seen in sections 2 and 3 how the orbifold Yang-Mills gauge theories with eight supercharges appear in matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^d$, we will now leave the semi-classical regime, and consider the full dynamics of the orbifold theories. In this section, following the strategy of [13], we will briefly discuss how the assumed correspondence with the heterotic string theory on tori leads to predictions about the non-perturbative dynamics of the corresponding orbifold Yang-Mills theories. As in [13], the expected properties of string theory suggest the existence of a multitude of fixed points in various dimensions.

To keep our discussion simple, we will avoid non-trivial supergravity backgrounds by assuming the symmetric distribution of D8-branes among the components of the singular locus.

5.1. Yang-Mills Dynamics on $\mathbf{S}^1 \times \mathbf{S}^1/\mathbf{Z}_2$

When the size of the space-time cylinder shrinks to zero volume, matrix theory on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$ is supposed to reproduce the Type I and heterotic $SO(32)$ string theory [4] in the light cone gauge. In particular, since these string theories exhibit $\mathcal{N} = 1$ super Poincaré invariance in ten dimensions, our matrix theory description should recover the full $SO(8)$ Lorentz invariance in the transverse space-time dimensions. In the orbifold Yang-Mills theory, only an $SO(7)$ subgroup is manifest. This situation has been analyzed in the related case with maximum supersymmetry [34,3], and it has been argued that in the appropriate limit, the theory grows another macroscopic space-time dimension. The space-like torus of the $2 + 1$ dimensional Yang-Mills theory can carry a non-zero magnetic flux, and its quanta behave as KK modes along an extra space-time dimension of radius

$$R' = (2\pi)^3 \frac{\ell_{11}^3}{R_1 R_2}. \quad (5.1)$$

In the orbifold case, the torus becomes a cylinder. The first interesting question to be asked is what is the behavior of the dynamically generated space-time dimension under the \mathbf{Z}_2 orbifold symmetry. Since the KK modes along the extra dimension correspond to the quanta of magnetic flux through that cylinder, they are even under the \mathbf{Z}_2 symmetry. Hence, the extra dimension is also even under the orbifold group, and the arguments [34,3] for the enhanced $SO(8)$ invariance in the IR carry over to the orbifold theory.

The correspondence with the heterotic and Type I $SO(32)$ string theory thus predicts the existence of a non-trivial infrared fixed point with eight supercharges in the Yang-Mills

gauge theory on the 2+1-dimensional orbifold. The supercharges are in the $\mathbf{8}_s$ of the global $SO(8)$ of R symmetries. The fixed point is associated with the superconformal group of the half-infinite 2+1 manifold $\mathbf{R}^2 \times \mathbf{R}/\mathbf{Z}_2$, with supercharges that respect chiral boundary conditions.

5.2. Yang-Mills Dynamics on $\mathbf{S}^1 \times \mathbf{T}^2/\mathbf{Z}_2$

In 3 + 1 dimensions, the dimensionless Yang-Mills coupling is given in terms of the space-time compactification parameters by

$$g^2 = (2\pi)^4 \frac{\ell_{11}^3}{R_1 R_2 R_3}. \quad (5.2)$$

The theory contains states with non-zero magnetic flux through the three two-dimensional cycles of the orbifold. As the volume of the space-time $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^2$ shrinks to zero, these modes become light, and can be naturally interpreted as KK modes along three macroscopic dimensions in space-time [17,18,13]. The magnetic fluxes through the $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{S}^1$ two-cycles are even under the \mathbf{Z}_2 orbifold group, and therefore correspond to space-time dimensions that are also even. On the other hand, the flux through the $\mathbf{T}^2/\mathbf{Z}_2$ cycle changes its sign under the \mathbf{Z}_2 action, and represents a KK mode along a space-time dimension which changes its orientation under the \mathbf{Z}_2 . As the volume of the original $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^2$ shrinks to zero, the macroscopic space-time again corresponds to M-theory compactified on $\mathbf{S}^1/\mathbf{Z}_2 \times \mathbf{T}^2$. As in the case with maximum supersymmetry [13], these two compactifications are related in the Yang-Mills theory by

$$g^2 \rightarrow \frac{(2\pi)^2}{g^2}. \quad (5.3)$$

The space-time T-duality between the two heterotic string compactifications predicts the existence of an electric-magnetic duality between the corresponding orbifold Yang-Mills theories with eight supercharges.

In principle, one would want to reconstruct in matrix theory the full U-duality group of the heterotic string vacua on \mathbf{T}^2 . A full analysis would require a more detailed understanding of orbifold Yang-Mills dynamics in the presence of generic space-time Wilson lines, and is clearly beyond the scope of this paper.

5.3. Yang-Mills Dynamics on $\mathbf{S}^1 \times \mathbf{T}^3/\mathbf{Z}_2$

Before orbifolding by \mathbf{Z}_2 , this case corresponds to matrix theory on a four-torus. The gauge coupling is

$$g^2 = (2\pi)^6 \frac{\ell_{11}^6}{\tilde{R}R_1R_2R_3R_4}; \quad (5.4)$$

the effective dimensionless coupling equals

$$\tilde{g}^2 \equiv \frac{g^2}{(\rho_1\rho_2\rho_3\rho_4)^{1/4}} = (2\pi)^4 \frac{\ell_{11}^3}{(R_1R_2R_3R_4)^{3/4}}. \quad (5.5)$$

As the space-time four-torus shrinks uniformly to zero volume, the gauge theory is accordingly expected to flow to a UV fixed point [13]. It was pointed out, however, that there are no known fixed points with the expected properties in $4 + 1$ dimensions. Instead, the proper description of the UV fixed point seems to be in terms of a $(0, 2)$ supersymmetric non-trivial fixed point in $5 + 1$ dimensions [35,13,36].

In terms of the $4 + 1$ dimensional theory, the extra dimension appears because the theory contains states that become light in the appropriate limit [35]. These states are given by marginal bound states of 4-dimensional Yang-Mills instantons, which represent solitonic particles in the $4 + 1$ dimensional theory. The state with instanton number Q carries energy

$$E = \frac{(2\pi)^2 Q}{g^2}, \quad (5.6)$$

which suggests that Q can be interpreted as the wave number in an extra dimension of radius

$$\rho' = (2\pi)^5 \frac{\ell_{11}^6}{\tilde{R}R_1R_2R_3R_4}. \quad (5.7)$$

Upon orbifolding one space-time dimension, we obtain a Yang-Mills theory on $\mathbf{S}^1 \times \mathbf{T}^3/\mathbf{Z}_2$. What is the behavior of the extra dimension under the orbifold group \mathbf{Z}_2 ? Since the orbifold group reflects three out of four dimensions of the torus, the instanton number is odd, therefore the extra dimension is odd under the \mathbf{Z}_2 orbifold group. The total \mathbf{Z}_2 acts on the product of the \mathbf{T}^3 and the extra dimension, and the theory becomes a $5 + 1$ dimensional field theory on the corresponding orbifold. String dynamics again suggests the existence of a non-trivial UV fixed point in this six-dimensional orbifold field theory with eight supercharges.

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